Birla Institute of Technology & Science, Pilani
Work Integrated Learning Programmes Division
M.S. Software Engineering at Wipro Technologies (WASE)
First Semester 2013 - 2014 (September 2013 to March 2014)
Compre Exam (Regular) - SOLUTIONS

Course Number : SEWP ZC472
Course Title : **COMPUTER GRAPHICS**
Type of Exam : Open **Book**
Weightage : 60 %
Duration : 3 hrs
Date of Exam :

No. of Pages : 2
No. of Questions : 12

Session : AN

Note:
1. Please read and follow all the instructions given on the cover page of the answerscript.
2. Start each answer from a fresh page. All parts of a question should be answered consecutively.
3. ALL QUESTIONS CARRY EQUAL MARKS

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1. In general, we say that the complexity of the image space approach to the hidden surface removal is proportional to the number of polygons. But performance studies have shown almost constant performance. Explain this result in detail.
   Solution: In image space approach we examine each pixel in the image to determine which face of which object should be displayed at that pixel. The complexity depends upon the number of faces and the number of pixels to be considered. So the complexity of the approach is proportional to the number of polygons.
   Suppose , consider ray casting model. Consider a ray that leaves the center of the projection and passes through a pixel. We can intersect this ray with each of the planes the ray passes through the polygon and finally for those rays ,find the intersection closest to the center of projection So here the fundamental operation is the intersection of rays with polygons. For an n x m display , we need to carry this operation nmk times,giving constant complexity. Complexity = O(pixels * objects) e.g. 1286*1024 pixels and 1 million polygons, Complexity = O(1.3 * 10^{12}).

2. Let $3x - 2y + 6z - 5 = 0$ be the equation of a plane viewed from eye position (1,1,1) ,while looking at the point (0,0,0). Is the front of the plane visible?

   Solution: No, It is a back face
   V = (1,1,1), N = (3,-2,6) (expect the clear explanation here)
   Because V.N = 1 > 0

3. Suppose that we divide a Bezier surface patch, first along the u – direction. Then in V, we only subdivide one of the two patches we have created. Whether this process creates a gap in resulting surface? If so find the solution to this difficulty.
   Solution: It creates a gap in the resulting surface (Explanation expected). For this we need to ensure the smooth transition from one section to the other by establishing both

zero order continuity and first order continuity at the boundary line.(how to achieve them to be explained)

4. Justify the statement "Sutherland – Hodgeman polygon clipping algorithm works for only convex clipping regions". If not support the statement "it is possible to apply this algorithm to clip concave polygons also".

   Solution: Convex polygons are correctly clipped by the S – H algorithm, but concave polygons may be displayed with extraneous lines.(An example Fig is expected for each)
   There are several things we could do to correctly display concave polygons. First one, we could split the concave polygon into two or more convex polygons and process each convex polygon separately.
   Another possibility is to modify the approach to check the final vertex list for multiple vertex points along any clip window boundary and correctly join pairs of vertex. Finally we could use a more general polygon clipper, such weiler – Atherton algorithm or the weiler algorithm.

5. Apply a suitable 3 D transformation matrix to a line joining (1,1,1) and (2,3,4) to align it to the positive z – axis and so that it originates from the origin.

Solution: $p_0(1,1,1)$ and $p_1(2,3,4)$

The line segment p0p1 is$(x_1-x_0)i +(y_1-y_0)j+(z_1-z_0)k$ and point $p_0(x_0,y_0,z_0)$ the equation is $x = x_0+(x_1-x_0)t$, $y = y_0+(y_1-y_0)t$, $z = z_0+(z_1-z_0)t$

$v= p_0p_1 = i+2j+3k$ and the equation is $x =1+t$, $y = 1+2t$, $z =1+3t$.

We perform the alignment through the following of transformations

a. Rotate about x-axis by an angle Θ so that the vector rotates into the upper half of the xz - plane.
b. Rotate the vector v about the y axis by an angle $-\Theta$ so that v rotates to the positive z-axis.
c. Calculate the inverse transformation that aligns the vector K with the vector v.

6. A solid tetrahedron is given by the point vectors A(1,1,1) , B(3,1,1) , C (2,1,3) and D(2,2,2) and a point light source is kept at P(2,3,4). Using back face detection method ,find the surfaces on which the light falls and the surfaces which are to be shadowed.

   Solution: Surfaces in tetrahedran are ABC,BCD, CDA,ADB

   From Back face detection method: V.N > 0, then backface

   V.N < 0, then frontface

   V.N = 0, then almost backface

   For ABC: Normal N = (A-B)x(B-C)x(C-A) = (-8,0,4)

   V.N = (2,3,4). (-8,0,4) = 0 i.e. surface ABC is almost backsurface,looks like a line

   For BCD: N = (0,3,-3)

   V.N = -3 < 0. i.e this is a front face

   For COA: N = (2,-5,1)

   V.N = -15 < 0.i.e this is a front face

   For ADB: N = (0,4,4)

V.N = 28>0 i.e.this is a back face

7. What happens in the Bezier curve if the values of the control points P_0 and P_1 are the same?

 Solution: It can be shown that $B(u) = p_0 + u(p_1 - p_0)$

 If p_1 and p_0 then $B(u) = p_0$ for all values of u.

 Then it generates the closed curve.

8. Consider a window having vertices A(1,1),B(5,3),C(4,5) and D(0,2). Find the normalization matrix that maps this window onto normalized co – ordinate system.

 Solution:

First rotate the angle about A so that it is aligned with the coordinate axes.

Next, Calculate the following

We form N by computing

a) A translation taking (xw_{min}, yw_{min}) to (xv_{min}, yv_{min})
b) A scaling about L(xvmin, yvmin)with
 a. $S_x = (xv_{max} – xv_{min}) / (xw_{max} – xw_{min})$
 b. $S_y = (yv_{max} – yv_{min}) / (yw_{max} – yw_{min})$

So $N = S_{Sx.\,Sy.\,L} \cdot T_v =$

Sx	0	(-sx . xwmin + xvmin)
0	Sy	(-Sy . ywmin + yvmin)
0	0	1

Finally apply rotation and the transformation N to find the required normalization.

Slope m = 3-1 / 5-1 = ½

We see that –Θ will be the direction of the rotation. The angle Θ is determined from the slope of a line by the equation tanΘ = 1/2 then

Sin Θ = 1/√5 Sin (-Θ) = - 1/√5

Cos Θ = 2/√5 Cos (-Θ) = -2/√5

The rotation matrix about A(1, 1) is then

$$R = \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & (1-1/\sqrt{5}) \\ -1/\sqrt{5} & 2/\sqrt{5} & (1-1/\sqrt{5}) \\ 0 & 0 & 1 \end{bmatrix}$$

The x-extent of the rotated window is the length of AB. Similarly y-extent is the length of AD.

D(A,B) = 2/√5 and d(A,D) = 1/√5

So N =

1/2/√5	0	-1/2/√5
0	-1/√5	-1/√5
0	0	1

The normalization transformation is then Nr = N . R

1/5	1/10	-1/10
-1/5	2/5	-1/5
0	0	1

9. Rotate a unit cube with centre at the origin about y – axis by 45^0.The viewer is at the infinity on z – axis and looks towards the origin. Then discuss the visibility of the faces.

Solution: Robertsalgorithm

The student should have to draw the unit cube with centre at origin

Then writing equations of all the faces. F1 = x – ½ = 0 etc

Then the complete volume matrix is

$$[v] = \left\{\begin{array}{cccccc} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ -1/2 & ½ & -1/2 & ½ & -1/2 & ½ \end{array}\right\}$$

Or this has to be modified after checking the correctness of the plane equations.

$$[V] \quad \begin{array}{cccccc} -2 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}$$

Since the cube is rotted by 45 degrees

$$[M_{45}{}^{0}] = \left\{\begin{array}{cccc} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right\}$$

Transformed volume matrix is $[V^1] = [M_{45}{}^{0}].[V]$

View direction vector E = [0   0   -1   0]

Now $[E].[V^1] = [-\sqrt{2} \quad \sqrt{2} \quad 0 \quad 0 \quad \sqrt{2} \quad -\sqrt{2}]$

SO F_1, F_6 are invisible faces, F_2, F_5 are visible faces, F_3, F_4 are parallel faces(expected to state the reason)

10. An incremental rotation about the z – axis can be represented by the matrix

$$\left(\begin{array}{cccc} 1 & -\theta & 0 & 0 \\ \theta & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

What negative aspects are there if we use this matrix for a large number of steps? Suggest a remedy for this.

Q: 3. An incremental rotation about the z-axis can be represented by the matrix.

$$\begin{bmatrix} 1 & -\theta & 0 & 0 \\ \theta & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

What negative aspects are there if we use this matrix for a large number of steps? Suggest a remedy for this.

Ans. → The determinant of the matrix is $1+\theta^2$.

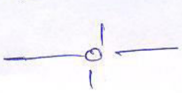
→ Repeated multiplications by this matrix increases the determinant, so the resulting operation becomes further and further from a rotation matrix causing the point to become further and further from the origin.

→ one remedy is to use the matrix:

$$R = \begin{bmatrix} 1 & -\theta & 0 & 0 \\ \theta & 1-\theta^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which has a determinant of 1.

→ Key: Consider points a distance of 1 from the origin.



11. Find the pixel color value at the centroid of the triangle with vertices (0, 3,5), (6,10,23) and (8,2,15). The color values at these points are 0, 12, 63 respectively.

Solution: Centre of the triangle (14/3, 5. 43/3)

Centre Pixel colour = 0+12+63/3 = 25

12. How a quadtree or octree could be used to speed up 2D or 3D picking in a openGL?

Quadtree is poupular data structure used for graphical representation of two dimensional digital pictures when the picture is a square matrix with 2^i dimension. Quad tree represents area where as octtrees are used for representing regions of space. Both of these have to be explained with example.

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